This work deals with three different constructions of the extended binary Golay code $\mathcal{G}_{24}$. The first construction is based on a projective plane of order four. In terms of it Steiner system $(5, 8, 24)$ is built. Linear span of its blocks forms a linear binary $[24, 12, 8]$ code $\mathcal{C}$. Every binary $[24, 12, 8]$ code is isomorphic to $\mathcal{C}$ which is known as extended binary Golay code $\mathcal{G}_{24}$. The second construction uses so-called Miracle Octad Generator (MOG). All MOG-words of weight eight form Steiner system $(5, 8, 24)$. The third construction uses impartial combinatorial game Mogul. In terms of its P-positions one can create a linear binary $[24, 12, 8]$ code. The fact that is is also a lexikographic code is useful for parametres estimate.